MAT 162—Exam #3—11/18/15

Name: ____

Show all work using correct mathematical notation. Calculators are not permitted.

1. (15 points) Find the sum of each series, or demonstrate that it diverges.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{3n+1}}{3^{2n+1}}$

   (b) $\displaystyle\sum_{n=3}^{\infty} \left(\sqrt{n} - \sqrt{n+1}\right)$

2. (10 points) Consider the sequence $a_n = \dfrac{3n^5+2}{4n^5+7}$. Evaluate

   (a) $\displaystyle\lim_{n\to\infty} a_n$

   (b) $\displaystyle\sum_{n=1}^{\infty} a_n$

3. (10 points) Find the limit of the sequence $a_n = n^{5/n}$. Show your work using correct limit notation.

4. (15 points) Consider the series $S = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$.

(a) Determine whether the series converges absolutely, converges conditionally, or diverges. Justify your answer using appropriate tests.

(b) Write out the fourth partial sum, S_4.

(c) How large must N be to ensure that the error in approximating S by the Nth partial sum S_N is at most 10^{-6}?

5. (25 points) Decide whether each series converges or diverges, and justify your conclusions using appropriate tests. You must give coherent arguments to receive credit.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3 + \sin n}{\sqrt{n^5 + 7n + 4}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(3n)!}{5^{2n}(n!)^3}$

6. (10 points) Consider the series $S = \sum_{n=1}^{\infty} a_n$, whose Nth partial sum is $S_N = \sum_{n=1}^{N} a_n$.

(a) Suppose that $S_N = 5 - \frac{1}{3^N}$ for all N. Find the sum of the infinite series S.

(b) Find a_3, the third term in the series.

7. (15 points) Find the radius and interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{2^n (x-5)^n}{\sqrt{n+1}}.$$

Justify your conclusions by citing appropriate tests.