MAT 162—Exam #2—3/27/15

Name: Solutions

Show all work using correct mathematical notation. Calculators are not allowed.

1. (15 points) Evaluate $\int x\cos(2x)\,dx$.

$u = x \qquad dv = \cos 2x \; dx$

$du = dx \qquad v = \frac{1}{2}\sin 2x$

$$\int x \cos 2x \, dx = \frac{1}{2} x \sin 2x - \int \frac{1}{2} \sin 2x \, dx$$

$$= \frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + C$$

2. (5 points) Which of the following is the correct form of the partial fraction decomposition for the function $f(x) = \dfrac{1}{x^3+4x}$?

(i) $\dfrac{A}{x} + \dfrac{B}{x^2+4}$

(ii) $\dfrac{A}{x} + \dfrac{B}{x+2} + \dfrac{C}{x-2}$

(iii) $\dfrac{A}{x} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$

(iv) $\dfrac{A}{x} + \dfrac{Bx+C}{x^2+4}$

(v) $\dfrac{A}{x} + \dfrac{B}{x+2} + \dfrac{Cx+D}{x^2+4}$

(vi) none of the above

3. (15 points) Evaluate $\int \tan^6 x \sec^4 x \, dx$.

$$\int \tan^6 x \sec^4 x \, dx = \int \tan^6 x \, (1 + \tan^2 x) \sec^2 x \, dx$$

Let $u = \tan x$

$du = \sec^2 x \, dx$

$$= \int u^6 (1 + u^2) \, du$$

$$= \int (u^6 + u^8) \, du$$

$$= \frac{1}{7} u^7 + \frac{1}{9} u^9 + C$$

$$= \frac{1}{7} \tan^7 x + \frac{1}{9} \tan^9 x + C$$

4. (15 points) Evaluate $\int \frac{x^2+1}{x^3+x^2} \, dx$.

$$\frac{x^2+1}{x^3+x^2} = \frac{x^2+1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$\Rightarrow \quad x^2 + 1 = Ax(x+1) + B(x+1) + Cx^2$$

$$= (A+C)x^2 + (A+B)x + B$$

$$\Rightarrow \begin{cases} A + C = 1 \\ A + B = 0 \\ B = 1 \end{cases} \Rightarrow A = -1, \quad C = 2$$

So

$$\int \frac{x^2+1}{x^3+x^2} \, dx = \int \frac{-1}{x} \, dx + \int \frac{1}{x^2} \, dx + \int \frac{2}{x+1} \, dx$$

$$= -\ln|x| - \frac{1}{x} + 2 \ln|x+1| + C$$

5. (15 points) Evaluate $\int_2^{\infty} e^{-3x}\, dx$.

$$= \lim_{b\to\infty} \int_2^b e^{-3x}\, dx$$

$$= \lim_{b\to\infty} -\frac{1}{3} e^{-3x} \Big|_2^b$$

$$= \lim_{b\to\infty} \left(-\frac{1}{3} e^{-3b} + \frac{1}{3} e^{-6} \right)$$

$$= \frac{1}{3} e^{-6}$$

6. (15 points) Evaluate $\int \frac{x^5}{\sqrt{4-x^2}}\, dx$.

Let $x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$\sqrt{4-x^2} = \sqrt{4(1-\sin^2\theta)}$

$= 2\cos\theta$

We get

$$\int \frac{32\sin^5\theta \cdot 2\cos\theta\, d\theta}{2\cos\theta}$$

$$= 32 \int \sin^5\theta\, d\theta$$

$$= 32 \int (1-\cos^2\theta)^2 \sin\theta\, d\theta$$

Let $u = \cos\theta$

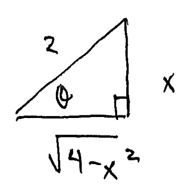
$du = -\sin\theta\, d\theta$

$$= -32 \int (1-u^2)^2\, du$$

$$= -32 \int (1-2u^2+u^4)\, du$$

$$= -32\left(u - \frac{2}{3}u^3 + \frac{1}{5}u^5\right) + C$$

$$= -32\left(\cos\theta - \frac{2}{3}\cos^3\theta + \frac{1}{5}\cos^5\theta\right) + C$$

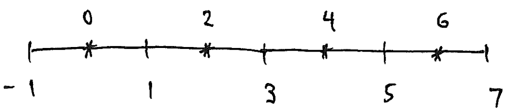


$$= -16\sqrt{4-x^2} + \frac{8}{3}(4-x^2)^{3/2} - \frac{1}{5}(4-x^2)^{5/2} + C$$

7. (15 points) Consider the integral $\int_{-1}^{7} e^{x^2}\, dx$.

(a) Write out the terms in the Midpoint Rule approximation M_4.

$$\Delta x = \frac{7-(-1)}{4} = 2$$



$$M_4 = 2\left(e^{0} + e^{4} + e^{16} + e^{36}\right)$$

(b) If $f(x) = e^{x^2}$, then it's easy to show that $|f''(x)| \leq 198e^{49}$ on $[-1, 7]$. Use this information to find an upper bound for the error in the approximation from part (a).

$$E_M \leq \frac{198e^{49} \cdot 8^3}{24 \cdot 4^2} = 264 e^{49}$$

(c) Which of the following best characterizes the quality of the estimate in part (a)?

pretty good OR terrible

8. (5 points) Consider the improper integral $\int_{1}^{\infty} \frac{x}{x^4 + 3x^2 + 1}\, dx$.

(a) Does the integral converge or diverge?

Converges

$$\frac{x}{x^4+3x^2+1} \leq \frac{x}{x^4} = \frac{1}{x^3}$$

(b) Which integral can you compare with to reach the above conclusion?

(i) $\int_{1}^{\infty} \frac{1}{x^3}\, dx$ (ii) $\int_{1}^{\infty} \frac{1}{x}\, dx$ (iii) $\int_{1}^{\infty} \frac{1}{x^4}\, dx$ (iv) $\int_{1}^{\infty} \frac{1}{x^{1/4}}\, dx$ (v) $\int_{1}^{\infty} \frac{1}{x^{3/4}}\, dx$

NOTE: There is no partial credit here; you must answer both parts correctly.