

MAT 162—Exam #2—3/27/15

Name: _____

Show all work using correct mathematical notation. Calculators are not allowed.

1. (15 points) Evaluate $\int x \cos(2x) dx$.

2. (5 points) Which of the following is the correct form of the partial fraction decomposition for the function $f(x) = \frac{1}{x^3 + 4x}$?

(i) $\frac{A}{x} + \frac{B}{x^2 + 4}$

(ii) $\frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$

(iii) $\frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

(iv) $\frac{A}{x} + \frac{Bx+C}{x^2+4}$

(v) $\frac{A}{x} + \frac{B}{x+2} + \frac{Cx+D}{x^2+4}$

(vi) none of the above

3. (15 points) Evaluate $\int \tan^6 x \sec^4 x \, dx$.

4. (15 points) Evaluate $\int \frac{x^2 + 1}{x^3 + x^2} \, dx$.

5. (15 points) Evaluate $\int_2^{\infty} e^{-3x} dx$.

6. (15 points) Evaluate $\int \frac{x^5}{\sqrt{4-x^2}} dx$.

7. (15 points) Consider the integral $\int_{-1}^7 e^{x^2} dx$.

(a) Write out the terms in the Midpoint Rule approximation M_4 .

(b) If $f(x) = e^{x^2}$, then it's easy to show that $|f''(x)| \leq 198e^{49}$ on $[-1, 7]$. Use this information to find an upper bound for the error in the approximation from part (a).

(c) Which of the following best characterizes the quality of the estimate in part (a)?

pretty good OR terrible

8. (5 points) Consider the improper integral $\int_1^{\infty} \frac{x}{x^4 + 3x^2 + 1} dx$.

(a) Does the integral converge or diverge?

(b) Which integral can you compare with to reach the above conclusion?

(i) $\int_1^{\infty} \frac{1}{x^3} dx$ (ii) $\int_1^{\infty} \frac{1}{x} dx$ (iii) $\int_1^{\infty} \frac{1}{x^4} dx$ (iv) $\int_1^{\infty} \frac{1}{x^{1/4}} dx$ (v) $\int_1^{\infty} \frac{1}{x^{3/4}} dx$

NOTE: There is no partial credit here; you must answer both parts correctly.