MAT 161—Exam #1—9/16/14

Name: Solutions

Calculators are NOT allowed. Show all work using correct mathematical notation.

1. (24 points) Evaluate each of the following limits, and enter your final answer in the box provided. For this problem, only your final answer will be graded, so there is no partial credit within each part.

(a) $\lim_{x \to 0} \frac{x^2 - 1}{x^2 + 2x - 3} = \boxed{1/3}$

$= \frac{0 - 1}{0 + 0 - 3}$

(b) $\lim_{x \to 2} \frac{3x - 6}{x^2 - 4} = \boxed{3/4}$

$= \lim_{x \to 2} \frac{3(x-2)}{(x-2)(x+2)}$

$= \frac{3}{4}$

(c) $\lim_{x \to 1} \frac{x - 3}{(x - 1)^2} = \boxed{-\infty}$

" $\frac{-2}{0^+}$ "

(d) $\lim_{x \to 0} \frac{\sin x}{x \cos 4x} = \boxed{1}$

$= \lim_{x \to 0} \frac{\sin x}{x} \lim_{x \to 0} \frac{1}{\cos 4x}$

$= 1 \cdot 1$

$= 1$

(e) $\lim_{x \to -\infty} \frac{x^2 + 4x + 5}{7x^2 + 3} = \boxed{1/7}$

$= \lim_{x \to -\infty} \frac{1 + 4/x + 5/x^2}{7 + 3/x^2}$

(f) $\lim_{x \to 0} x^8 \sin(1/x^2) = \boxed{0}$

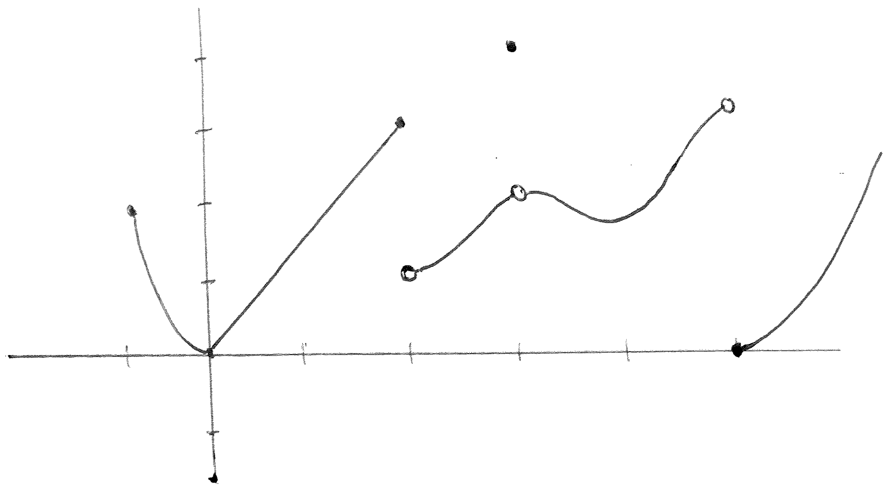
by Squeeze Thm since

$-x^8 \le x^8 \sin(1/x^2) \le x^8$

$\downarrow$ $\downarrow$

0 0

2. (28 points) Consider the function $y = f(x)$ graphed below.



Evaluate each of the following quantities. If a quantity does not exist, say so. Explanations are not required, and there is no partial credit within each part.

(a) $\lim_{x\to 2^+} f(x)$ = 1

(b) $f(2)$ = 3

(c) $\lim_{x\to 3} f(x)$ = 2

(d) $\lim_{x\to 5^-} f(x)$ = 3

(e) $\lim_{x\to 5} f(x)$ does not exist

(f) The instantaneous rate of change of f at $x = 1$

$\frac{3}{2}$ (slope of the tangent line)

(g) The average rate of change of f on the interval $[-1, 2]$.

$$\frac{f(2) - f(-1)}{2 - (-1)} = \frac{1}{3}$$

3. (20 points) Consider the function

$$f(x) = \begin{cases} x^3 - 1 & \text{if } x < 2 \\ x^2 + 6 & \text{if } 2 \leqslant x < 5 \\ 6x + 1 & \text{if } x \geqslant 5 \end{cases}.$$

Determine each of the following. If a quantity does not exist, say so. There is no partial credit within each part.

(a) $\lim_{x \to 2^-} f(x)$

$= \lim_{x \to 2} (x^3 - 1) = 7$

(b) $\lim_{x \to 2^+} f(x)$

$= \lim_{x \to 2} (x^2 + 6) = 10$

(c) $f(2)$

$= 10$

(d) $\lim_{x \to 5} f(x)$

$= 31$

(e) The value(s) of x at which f fails to be continuous

$x = 2$

4. (6 points) An object's position along a line is given by the formula $s(t) = t^3$, where s is measured in meters and t is measured in seconds. Find the object's average velocity over the interval $0 \leqslant t \leqslant 2$.

$$\frac{s(2) - s(0)}{2 - 0} = \frac{2^3 - 0^3}{2} = 4 \text{ m/s}$$

5. (5 points) Suppose that $\lim_{x \to 7} f(x) = 3$. Evaluate $\lim_{x \to 7} \frac{x\cos(\pi x)}{[f(x)]^2}$.

$$\lim_{x \to 7} \frac{x \cos(\pi x)}{[f(x)]^2} = \frac{7 \cos(7\pi)}{3^2} \quad \text{by Limit Laws}$$

$$= -\frac{7}{9}$$

6. (5 points) According to the Intermediate Value Theorem, in which of the following intervals is the equation $x^3 - 3x + 1 = 0$ guaranteed to have a solution? There may be more than one answer–circle all that apply!

(i) $[-2,-1]$ (ii) $[-1,0]$ (iii) $[0,1]$ (iv) $[1,2]$ (v) $[2,3]$

Let $f(x) = x^3 - 3x + 1$.

$f(-2) = -1$ $f(0) = 1$ $f(2) = 3$

$f(-1) = 3$ $f(1) = -1$ $f(3) = 19$

7. (12 points) Evaluate $\lim_{x \to 3} \frac{x^2 - x - 6}{\sqrt{x+1} - 2}$, showing all steps **using correct limit notation**.

$$= \lim_{x \to 3} \frac{(x-3)(x+2)}{\sqrt{x+1} - 2}$$

$$= \lim_{x \to 3} \frac{(x-3)(x+2)}{\sqrt{x+1} - 2} \cdot \frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2}$$

$$= \lim_{x \to 3} \frac{(x-3)(x+2)(\sqrt{x+1} + 2)}{(x+1) - 4}$$

$$= \lim_{x \to 3} (x+2)(\sqrt{x+1} + 2) = 20$$